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Inhalt

1	Fourier–Analysis	1
1.1	Motivation: Ein erstes Beispiel	1
1.2	Approximation im quadratischen Mittel	7
1.2.1	Skalarprodukte und Hilbert–Räume	7
1.2.2	Orthogonale Systeme	12
1.2.3	Bestapproximation und Konvergenz im quadratischen Mittel	15
1.3	Fourier–Reihen	18
1.3.1	Definition der trigonometrischen Fourier–Reihen	19
1.3.2	Konvergenz im quadratischen Mittel	22
1.3.3	Punktweise Konvergenz	24
1.3.4	Abklingverhalten der Fourier–Koeffizienten	33
1.3.5	Das Gibbs’sche Phänomen	36
1.3.6	Diskrete Fourier–Transformation (DFT)	40
1.3.7	Schnelle Fourier–Transformation (FFT)	43
1.3.8	Technische Durchführung der FFT	46
1.3.9	Beliebige Perioden	53
1.3.10	Mehrdimensionale Fourier–Transformation	54
1.3.11	Einige Anwendungen	56
1.4	Die Fourier–Integraltransformation	63
1.4.1	Motivation	63
1.4.2	L_1 –Theorie	64
1.4.3	L_2 –Theorie	72
1.4.4	Distributionen und ihre Fourier–Transformation	78
1.4.5	Die Unschärferelation	82
1.4.6	Zusammenfassung	87
2	Sturm–Liouville–Probleme	89
2.1	Das Eigenwertproblem	89
2.2	Reguläre Sturm–Liouville–Probleme	104
2.3	Inverse Sturm–Liouville–Operatoren	107

2.4	Anwendungen	112
3	Komplexe Funktionen	117
3.1	Differenzierbarkeit in \mathbb{C}	117
3.2	Konturintegrale in \mathbb{C}	120
3.3	Der Cauchysche Integralsatz	126
3.4	Potenzreihen	131
3.5	Der Residuenkalkül	136
4	Laplace–Transformation	145
4.1	Strukturelles: Existenz und Eindeutigkeit	145
4.2	Rechenregeln	152
4.3	Rücktransformation	158
4.3.1	Vereinfachungen über Rechenregeln	158
4.3.2	Systematische Rücktransformation	161
4.3.3	Numerische Rücktransformation	168
4.4	Anwendung: die „Systemtheorie“ der Ingenieure	172

Literatur

Die Vorlesung baut nicht streng auf irgendeinem Buch auf, sondern geht ihren eigenen Weg. Die angegebenen Referenzen dienen dazu, sich *unabhängig* vom Skript entsprechende Grundlagen anzueignen oder spezielle Inhalte zu vertiefen. Es handelt sich um eine recht willkürliche Auswahl: Neben den angegebenen Büchern gibt es sicherlich jede Menge weiterer Literatur, die den behandelten Stoff analog abdeckt.

Zu Kapitel 1 (Fourier–Analysis):

- 1) H. DYM AND H. P. MCKEAN : *Fourier Series and Integrals*, Academic Press, 1972
This book contains numerous applications of Fourier analysis. Strongly recommended for anyone who is interested in applications and wants to deepen their understanding of Fourier analysis. It also includes a nice description of Lebesgue integration and group theory.
- 2) T. W. KÖRNER : *Fourier Analysis*, Cambridge University Press, 1988
This is a monumental work on Fourier analysis, consisting of a bunch of interrelated essays. Read one section per day! You will gain a lot. Highly recommended.
- 3) J. S. WALKER : *Fourier Analysis*, Oxford University Press, 1988
A well-written and solid book on Fourier analysis with applications on optics, computer-aided tomography, spherical harmonics, etc.
- 4) G. B. FOLLAND : *Fourier Analysis and Its Applications*, Brooks/Cole Publishing Co., 1992
An introductory but extremely well-written textbook on Fourier analysis. Contains chapters on special functions, generalized functions (distributions), and Greens functions. Applications are mainly for differential equations. Expensive but worth buying it.
- 5) J. M. ASH (ED.) : *Studies in Harmonic Analysis*, Mathematical Association of America, 1976

This is a collection of conference talks by the authorities held in Chicago in 1975. Most of the chapters are as if these authorities are directly talking to you in a friendly manner about the essence of the ideas in harmonic analysis without much detailed proofs. Contains really deep mathematics.

- 6) S. G. KRANTZ : *A Panorama of Harmonic Analysis*, Mathematical Association of America, 1999

This book gives a historical perspective of harmonic analysis ranging from classical to modern, from elementary to advanced. One can see how subtle it is to sum multiple Fourier series. This also includes short description on wavelets. Highly recommended.

- 7) E. M. STEIN AND G. WEISS : *Introduction to Fourier Analysis on Euclidean Spaces*, Princeton University Press, 1971 A classic of the multidimensional Fourier analysis. Includes detailed discussions on the invariance properties of Fourier transform.

- 8) A. ZYGMUND : *Trigonometric Serie*, (2nd Ed., Volume I & II combined), Cambridge University Press, 1959

An ultimate bible on Fourier series and integrals for hard analysts. This is basically a dictionary. Almost no applications are treated here.

- 9) R. N. BRACEWELL : *The Fourier Transform and Its Application*, (2nd Ed., Revised), MacGraw-Hill, 1986

Another bible for engineers. Contains an excellent pictorial dictionary of many functions and their Fourier transforms.

- 10) G. P. TOLSTOV : *Fourier Series*, Dover, 1972.

The most cost effective book (about \$12). Very well written. Highly recommended.

- 11) G. H. HARDY AND W. W. ROGOSINSKI : *Fourier Series*, Dover, 1999.

This is a prelude to Zygmund's book. Spirit of pure mathematics. No applications included. Economical (\$7).

- 12) W. L. BRIGGS AND V. E. HENSON : *The DFT: An Owner's Manual for the Discrete Fourier Transform*, SIAM 1995

This is a very useful book on DFT. Includes many practical applications, such as tomography, seismic migrations, difference equation solvers. Detailed analysis on the error of the DFT. A nice book to keep on your desk.

- 13) A. TERRAS : *Fourier Analysis on Finite Groups and Applications*, Cambridge University Press, 1999.

Another type of Fourier analysis. A more detailed version of the first half

of Chapter 4 of Dym and McKean plus many more examples and applications of that aspect of Fourier analysis.

Zu Kapitel 2 (Sturm–Liouville–Probleme):

[WW] WOLFGANG WALTER, *Gewöhnliche Differentialgleichungen*, Springer, 1996.

[Tol] G. P. TOLSTOV : *Fourier Series*, Dover, 1972.
Man beachte hier speziell das letzte Kapitel.

Zu Kapitel 3 (Komplexe Funktionen):

[Jän] KLAUS JÄNICH: *Funktionentheorie – Eine Einführung*, Springer, 1999.

Zu Kapitel 4 (Laplace–Transformation):

[Föl] OTTO FÖLLINGER, *Laplace–, Fourier– und z –Transformation*, Hüthig Verlag, 2000.

[Mar] JERROLD E. MARSDEN, *Basic Complex Analysis*, Freeman 1987.

[BR] R.E. BELLMANN AND R.S. ROTH, *The Laplace Transform*, World Scientific 1984.

[Doe] GUSTAV DOETSCH, *Einführung in Theorie und Anwendung der Laplace–Transformation*, Birkhäuser 1970.

[SGV] W. STRAMPP, V. GANZHA, E. VOROZHTSOV, *Höhere Mathematik mit Mathematica 4*, Vieweg, 1997.